Mathematics 181D — Spring 2008

Exam 4

April 17, 2008

KEY

Technology used: ______________________ **Only write on one side of each page.**

- Show all of your work. Calculators may be used for numerical calculations and answer checking only.

Potential Problems from 8.1 to 8.6

1. [5, 5, 5 points] Determine whether the following sequences converge or diverge. Find the limit of each convergent sequence. Explain your answer.

   (a) $a_n = \frac{(-1)^{n+1}}{\sqrt{2n-1}}$: **Solution:** $\frac{-1}{\sqrt{n}} \leq \frac{(-1)^{n+1}}{\sqrt{2n-1}}. \leq \frac{1}{\sqrt{n}}$ for all positive integers n and $\frac{1}{\sqrt{n}} \to 0$ as $n \to \infty$.

   (b) $a_n = \left(1 - \frac{1}{n}\right)^{5n} = \left[\left(1 - \frac{1}{n}\right)^n\right]^5$ which limits to $\left[e^{-1}\right]^5$ by Part 5 of Theorem 5 on page 509.

   (c) $a_n = \sqrt[n]{n^5} = [\sqrt[n]{n}]^5$ which limits to $1^5 = 1$ by Part 2 of Theorem 5 on page 509.

2. [10 points] Do **one** (1) of the following.

   (a) Use partial fractions to find the exact sum of the following series.

$$\sum_{n=1}^{\infty} \frac{6}{(2n-1)(2n+1)}$$

   Solution: $\frac{6}{(2n-1)(2n+1)} = \frac{3}{2n-1} - \frac{3}{2n+1}$ so $s_1 = [3-1]$, $s_2 = [3-1] + \left[1 - \frac{3}{5}\right]$, $s_3 = [3-1] + \left[1 - \frac{3}{5}\right] + \left[\frac{3}{5} - \frac{3}{7}\right]$ and, in general and after telescoping, $s_n = 3 - \frac{3}{2n+1.}$ Hence the series converges to $\lim_{n\to\infty} s_n = 3 - 0 = 3$.

   (b) Find the values of x for which the following geometric series converges. In addition, find the sum of the series (as a function of x) for those values of x.

$$\begin{aligned} \sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^{n-1} (x-3)^{n-1} &= \sum_{n=1}^{\infty} 1 \left(\frac{x-3}{2}\right)^{n-1} \\ &= \frac{1}{\left(\frac{x-3}{2}\right)} = \frac{2}{x-3} \text{for all numbers satisfying } \left|\frac{x-3}{2}\right| < 1 \\ \text{That is for } 1 &< x < 5. \end{aligned}$$

   (c) **HW:** Use a geometric series to express the number $0.\overline{7} = 0.77777777\cdots$ as the ratio of two integers. [Hint: this number is (better and better) approximated by 0.7, 0.77, 0.777, etc.]
   Solution: $0.\overline{7} = \sum_{n=1}^{\infty} \frac{7}{10} \left(\frac{1}{10}\right)^{n-1} = \frac{7/10}{1-1/10} = \frac{7}{9}$

3. [15 points] Do **one** (1) of the following.

(a) Use the integral test to determine if the following series converges or diverges. Give reasons and show your work. You do not need to prove that the appropriate function is decreasing.

$$\sum_{n=1}^{\infty} \frac{e^{-n}}{1+e^{-2n}}$$

(b) The P-series $\sum_{n=1}^{\infty} \frac{1}{n^3}$ can be shown to converge using the integral test. Bound the error in using $S_4 = 1 + \frac{1}{2^3} + \frac{1}{3^3} + \frac{1}{4^3} = \frac{2035}{1728}$ to approximate the actual limit of this infinite series. (Use the error bound for the integral test.)

4. [15 points each] Do **four** (4) of the following but make sure **at least one** (1) of them is an alternating series.

Which of the following series converge absolutely, which converge conditionally, and which diverge? Give reasons and show your work.

(a) **HW:** $\sum_{n=1}^{\infty} \frac{1}{1+\ln(n)}$ Compare to $\sum \frac{1}{n}$ – diverges

(b) $\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!}$: Use ratio test – converges.

(c) $\sum_{n=1}^{\infty} (-1)^{n+1} \left(\sqrt[n]{10}\right)$:AST fails but diverges by Nth Term test

(d) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{\sqrt{n}}{n^2+n}$: Converges absolutely by comparison to $\sum \frac{1}{n^{3/2}}$

(e) **HW:** $\sum_{n=1}^{\infty} \left(1 - \frac{3}{n}\right)^n$: diverges by N'th Term Test

(f) $\sum_{n=1}^{\infty} \frac{(-3)^n}{n!}$: converges absolutely by Ratio Test

(g) **HW:** $\sum_{n=2}^{\infty} \frac{n}{(\ln(n))^n}$: Converges by Root Test